Quasiattractor in models of new and chaotic inflation

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Inflation with a scalar-field potential of the form $\lambda(\phi^2 - v^2)^2$ can be described in terms of a parametrical attractor with critical points, whose driftage depends on the control value of the slowly changing Hubble rate. The method allows us to easily obtain theoretical expressions for fluctuations of inhomogeneity in both the cosmic microwave background and distribution of matter. We find the region for admissible values of potential parameters, wherein theoretical predictions are consistent with experimental results within the limits of measurement uncertainties.

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I. INTRODUCTION

At present, in cosmology there is a problem in determining the parameters of inflation, which, in fact, has become the standard model for the early stage of Universe evolution [1, 2, 3, 4, 5] *before* the Big Bang. The Big Bang is considered now as a short stage of reheating of the Universe due to a transformation of inflaton energy into the energy of matter, whereas the inflaton is usually ascribed to be a real scalar field. In this respect it would be useful to have a complete arsenal of effective methods in order to describe various characteristics at the inflationary stage. These instruments would allow us to carry out a more thorough analysis of theoretical models in comparison with quite precise modern experimental data. At present, the basic tool of such studies is the slow-roll approximation in the field equations of inflation (see the review in [5]).

The slow-roll dynamics of evolution can be consistently treated in the framework of a $1/N$-expansion at a large amount of e-folding N for the scale factor of expansion, which was presented in [6] as a general analysis of relative scaling behavior of inflaton quantities versus $1/N$. So, the inflaton potential V gets the characteristic scale M by $V \sim N\, M^4$ at $M \sim 10^{16}$ GeV, while the inflaton field ϕ behaves like $\phi \sim \sqrt{N}\, M_{\rm Pl}$ with $M_{\rm Pl}$ being the Planck mass as given by the Newton gravitational constant $G = 1/M_{\rm Pl}^2$. In this respect, one could expect, for instance, a characteristic value of quartic coupling in the inflaton self-action of the order of $\lambda \sim 1/N \left(M/M_{\rm Pl}\right)^4 \sim 10^{-14}$. Thus, one gets the tool of strict consideration of the slow-rolling regime.

We follow another way, which is the method of the quasiattractor. This approach was offered in [7] for the case of a quadratic potential, in order to generalize and develop investigations considering the dependence of cosmological evolution on initial data that were performed in [8, 9, 10, 11, 12]. Further, we have applied the same approach to the potential of $\lambda\phi^4$ in [13]. This kind of potential refers to the models of "chaotic inflation", when the evolution occurs from large fields at Planck scales towards the global minimum at $\phi = 0$. However, it would be useful to somehow generalize these results to a potential of the form $\lambda(\phi^2 - v^2)^2$, permitting, first, the opportunity of a situation with the scenario of "new inflation", when the field evolves from a position in the vicinity of a local maximum at zero value of the field to the global minimum at $\phi = v$. Second, as we will see, such a potential allows us to essentially expand the region of admissible values of the potential parameters consistent with the data. This fact significantly increases the viability of the model.

The quasiattractor approach can be described by the following: After choosing the model potential we derive the equations of the system motion, which are generally not analytically soluble, so that we try to treat the problem by applying some consistent approximations in order to describe the system evolution. In the method of the quasiattractor we introduce new dimensionless variables with presumed properties of scaling. Then, the differential equations of the first order can be considered as an autonomous system. The system could attain stable critical points on a phase plane. The trajectories converge to these points, being the attractors. Our first task is to search for such critical points. The notion of "quasiattractor" refers to the stable critical point of an autonomous system[1] with external

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[1] The exact attractor arises at quite definite fixed functional forms of potential: at zero cosmological constant it is the exponent [14, 15,

parameters slowly drifting with the evolution. The position of the critical point is not fixed, since it is determined by the control parameters, which evolve and displace the critical point. But the evolution velocity of the control parameters is slow enough in order to consider the displacement of the point in the phase space as driftage. Thus, the system motion is the following: The system very quickly "falls" to the quasiattractor in the phase space, i.e. to the stable critical point slowly drifting during the evolution. So, the information about the initial position of the system is lost, while values of control parameters determining the position of attractor are important. The further evolution of system is exclusively determined by the driftage of the quasiattractor. The system motion is appropriated by the evolution of control parameters, and the system seems to lose some degrees of freedom.

As we will derive below, the driftage of the attractor is equivalent to the slow-roll regime of inflation treated in the framework of $1/N$-expansion considered in [6, 19].

In Section II we consider mathematical aspects of the quasiattractor, while in Section III we compare theoretical results with experimental data. Our results are in agreement with the precise analysis of a complete data set previously performed in [20, 21] in the framework of Monte Carlo Markov Chains. In the Conclusion we discuss the results obtained.

II. MATHEMATICAL ASPECTS

A. Equations of Motion

Let us consider the action of the inflaton in the form

$$S = \int dx^4 \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right\} \tag{1}$$

with the potential

$$V = \frac{\lambda}{4}(\phi^2 - v^2)^2. \tag{2}$$

The evolution of a homogeneous isotropic flat Universe is described by a Friedmann-Lemaitre-Robertson-Walker metric (FLRW) in Cartesian coordinates

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2(t), -a^2(t), -a^2(t)), \tag{3}$$

where $a(t)$ is the scale factor with its usual physical interpretation. The evolution equations read off as

$$\ddot{\phi} = -3H\dot{\phi} - \lambda\phi(\phi^2 - v^2), \tag{4}$$

$$\dot{H} = -4\pi G \dot{\phi}^2. \tag{5}$$

Here the dot denotes the derivative with respect to time t. The Hubble rate is defined by $H = \dot{a}/a$.

The Friedmann relation is derived from (4) and (5), so that

$$H^2 = \frac{4\pi G}{3} \left\{ \dot{\phi}^2 + \frac{1}{2}\lambda(\phi^2 - v^2)^2 \right\}. \tag{6}$$

All of the equations (4)–(6) are consequences of General Relativity.

B. Autonomous System

For the sake of simplicity let us change variables:

$$x = \frac{\kappa}{\sqrt{6}} \frac{\dot{\phi}}{H}, \tag{7}$$

16, 17], while at nonzero cosmological constant it is the hyperbolic cosine [18].

$$y = \sqrt[4]{\frac{\lambda}{12}}\frac{\sqrt{\kappa}}{\sqrt{H}}\sqrt{|\phi^2 - v^2|}, \tag{8}$$

$$z = \frac{\sqrt[4]{3\lambda}}{\sqrt{\kappa H}}, \tag{9}$$

$$u = \frac{\kappa v}{\sqrt{6}}, \tag{10}$$

where $\kappa^2 = 8\pi G$. Then, Eqs. (4)–(6) take the form[2]

$$x' = 3x^3 - 3x - 2y^2 z\sqrt{y^2 + u^2 z^2}, \tag{11}$$

$$yy' = \frac{3}{2}x^2 y^2 + xz\sqrt{y^2 + u^2 z^2}, \tag{12}$$

$$z' = \frac{3}{2}x^2 z, \tag{13}$$

where the prime denotes the derivative with respect to $N = \ln(a/a_{\text{init.}})$. Then the relation $\frac{\partial}{\partial t} = H\frac{\partial}{\partial N}$ is valid. The physical sense of N is that it counts the amount of e-folding during the expansion of the Universe from $t_{\text{init.}}$ till the current point, i.e. when the scale factor increases by e^N times.

In terms of the new variables the Friedmann relation reads off as

$$x^2 + y^4 = 1. \tag{14}$$

The equations are simplified, since they are already differential equations of the first order, though they are nonhomogeneous, which are easier for analysis than the initial ones.

Indeed, the two equations of (11) and (12) can be considered as an autonomous system of differential equations of the first order with external parameter z. Then, there is a question of the stability of given system. The numerical analysis shows, that the system is stable under some definite conditions. The control parameter of autonomous system is the slowly varying quantity z. Obviously, the driftage proceeds smoothly at $x^2 z \ll 1$. The question is when will the critical point be stable? Then, all of trajectories will approach this point, and it becomes the parametrical attractor, i.e. the quasiattractor, while the system, gradually having come to it, will remain at the critical point and drift together with it, and the evolution of the actually stable point (the quasiattractor) will be determined by the control quantity z.

Equations for the quasiattractor ($x' = y' = 0$) are the following:

$$3x^3 - 3x - 2y^2 z\sqrt{y^2 + u^2 z^2} = 0, \tag{15}$$

$$\frac{3}{2}xy^2 + z\sqrt{y^2 + u^2 z^2} = 0. \tag{16}$$

It is worth noticing that the system of equations is compatible with the Friedmann condition.

For the sake of simplification of system, we can make the following change of variable:

$$Y^2 = y^2 + u^2 z^2. \tag{17}$$

Then, the equations transform and look less cumbersome without radicals:

$$x' = 3x^3 - 3x - 2Yz(Y^2 - u^2 z^2), \tag{18}$$

$$Y' = \frac{3}{2}x^2 Y + xz, \tag{19}$$

$$z' = \frac{3}{2}x^2 z, \tag{20}$$

$$1 = x^2 + (Y^2 - u^2 z^2)^2. \tag{21}$$

[2] For brevity of notation, we take square roots in the arithmetic sense that corresponds to the case when the field takes values greater than the vacuum expectation, $|\phi| > v$, so that the scenario of chaotic inflation is realized. Otherwise, in the case of new inflation with $|\phi| < v$, one should take the root with the opposite sign under the substitution $y^2 \to -y^2$ in the radicand. This procedure is equivalent to removing the absolute value of the radicand in expression (8), so that y^2 can formally run to negative values in the model of new inflation.

The scaling properties of Y and y are equivalent, since these quantities differ by a shift, which depends on the external parameter controlling the driftage.

If the variable y is eliminated from the system, the equations for the critical point in the physical case of $x \neq 0$, $y \neq 0$ are reduced to the single equation in x (we recall that the quantity z is the parameter)

$$\frac{3}{2} x\sqrt{1-x^2} + z\sqrt{\sqrt{1-x^2} + u^2 z^2} = 0. \tag{22}$$

C. Analysis of the System

Let us analyze the stability of the critical point $(x_c,\, y_c)$. Introduce small deviations from the attractor $(\delta x,\, \delta y)$, then $x = x_c + \delta x,\; y = y_c + \delta y$. We obtain the following differential equations for the deviations in the linear approximation:

$$\begin{pmatrix} \delta x' \\ \delta y' \end{pmatrix} = \begin{pmatrix} 9x_c^2 - 3 & \dfrac{4z^2(3y_c^2 + 2u^2 z^2)}{3x_c y_c} \\ \dfrac{3}{2}\, x_c y_c & \dfrac{9x_c^2 y_c^4 + 4u^2 z^4}{6y_c^4} \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix}. \tag{23}$$

The Friedmann condition requires

$$x_c \delta x + 2y_c^3 \delta y = 0, \tag{24}$$

or

$$\begin{pmatrix} x_c & 2y_c^3 \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = 0, \tag{25}$$

i.e. the solution, satisfying the Friedmann equation, will be proportional to the eigenvector

$$v \sim \begin{pmatrix} 2y_c^3 \\ -x_c \end{pmatrix}. \tag{26}$$

Such an eigenvector for the given matrix exists, and is single with the eigenvalue

$$\mathcal{B} = 3 - 6y^4 - \frac{2}{3}\frac{z^2}{y^2}. \tag{27}$$

Therefore, the evolution goes according to the law

$$\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = C \begin{pmatrix} 2y_c^3 \\ -x_c \end{pmatrix} e^{\mathcal{B} N}. \tag{28}$$

Thus, we see, that this is the only solution of system. It satisfies the imposed constraints. Further advancement of the analysis of the autonomous system will consist in the direct examination of the stability of the obtained solution.

We require $\mathcal{B} < 0$ for stability of attractor. This condition is valid at small values of x and z (then, according to the Friedmann equation y^4 is close to 1) and $\mathcal{B}$ is certainly less than zero. The constraint on the smallness of x and z is actually valid, as we will see below.

D. The Universe Inflation

Let us consider Universe inflation due to the inflaton with the chosen potential. The condition of accelerated expansion is the following:

$$\ddot{a} > 0 \quad \Rightarrow \quad \frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Rightarrow \quad \frac{\dot{H}}{H^2} = -3x^2 > -1. \tag{29}$$

Accordingly, such an expansion regime ends up with

$$x_{\rm end}^2 = \frac{1}{3}, \tag{30}$$

$$y_{\rm end}^4 = \frac{2}{3}, \tag{31}$$

$$z_{\rm end}^2 = \frac{\sqrt{3u^2+1}-1}{u^2\sqrt{6}}. \tag{32}$$

During the actual process of expansion with acceleration, the quantities should satisfy the inequalities,

$$x_c^2 < x_{\rm end}^2, \tag{33}$$

$$y_c^4 > y_{\rm end}^4, \tag{34}$$

$$z^2 < z_{\rm end}^2. \tag{35}$$

One can see that such values are in agreement with the condition making the attractor stable ($\mathcal{B} < 0$), since (34) gives

$$\mathcal{B} < -1 - \frac{2}{3}\frac{z^2}{y^2}.$$

hence, the accelerated expansion is governed by the stable quasiattractor.

E. Characteristics of The Universe Expansion

First of all, we determine how many times the Universe expands from the initial state marked by "in" to the end of inflation marked by "end". The total amount of e-folding N is given by following expression, which follows from the equation for the parameter z in (13),

$$N_{\rm total} = \frac{2}{3}\int_{z_{\rm in}}^{z_{\rm end}} \frac{dz}{x_c^2 z}. \tag{36}$$

Here, the parameter x is set at the point of the quasiattractor.

In order to find numerical values of the theory parameters, one should compare it with observational data. Experiment measures the inhomogeneity of the cosmic microwave background, related to the inhomogeneity of matter, also independently measured, hence we need to find the distribution of the inflaton inhomogeneity, which leads to the matter inhomogeneity at the stage of reheating. Such inhomogeneity is given by the quantum fluctuations of the inflaton. Then, the spectral density of scalar and tensor perturbations look as

$$P_S(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot\phi}\right)^2 = \frac{\lambda}{8\pi^2}\,\frac{1}{x_c^2 z^4} \tag{37}$$

and

$$P_T(k) = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 = \frac{6\lambda}{\pi^2}\,\frac{1}{z^4}, \tag{38}$$

where the wave vector k is determined by the Hubble rate at the exit of fluctuations from the horizon, i.e. at $k = aH$.

Consider the ratio r determining the relative contribution of tensor spectrum,

$$r = \frac{P_T(k)}{P_S(k)} = 48\, x_c^2, \tag{39}$$

and define the spectral index n_S as

$$n_S - 1 \equiv \frac{d\ln P_S}{d\ln k}. \tag{40}$$

One can easily see that

$$\ln\frac{k}{k_{\rm end}} = N - 2\ln\frac{z}{z_{\rm end}}, \tag{41}$$

so that differentiation with respect to the wave vector is reduced to derivative with respect to the parameter z, determining the dynamics in the method of the quasiattractor.

F. Finding the Total N

From equation (22) at $x^2 \ll 1$ one approximately gets[3]

$$x_c^2 \approx \frac{4}{9} z^2 (1 + u^2 z^2). \tag{42}$$

The Friedmann condition in the forms of (6) and (14) strictly holds and yields $y^2 \approx 1$ in the limit under consideration, while the attractor position of (15), (16) reduced to (42) gives the slow-roll equation $3H\dot{\phi} + \partial V / \partial \phi = 0$.

Then, substituting the above expression into (36), we obtain

$$\begin{aligned} N_{\text{total}} &\approx \frac{3}{4} \left(u^2 \ln \frac{1 + u^2 z^2}{u^2 z^2} - \frac{1}{z^2} \right) \Big|_{z_{\text{in}}}^{z_{\text{end}}} \\ &= \frac{3}{4} \left(u^2 \ln \frac{\sqrt{3u^2+1} - 1 + \sqrt{6}}{\sqrt{3u^2+1} - 1} - \frac{u^2 \sqrt{6}}{\sqrt{3u^2+1} - 1} + \frac{1}{z_{\text{in}}^2} - u^2 \ln \frac{1 + u^2 z_{\text{in}}^2}{u^2 z_{\text{in}}^2} \right). \end{aligned} \tag{43}$$

To simplify this bulky expression we can separate out the function obtained by substituting z_{end}, giving the term,

$$F(u) = \frac{3}{4} \left(u^2 \ln \frac{\sqrt{3u^2+1} - 1 + \sqrt{6}}{\sqrt{3u^2+1} - 1} - \frac{u^2 \sqrt{6}}{\sqrt{3u^2+1} - 1} \right). \tag{44}$$

This function monotonically decreases in the interval $u \in [0, +\infty)$, hence, it is restricted by limits at the borders

$$\frac{3}{4} \leqslant F(u) \leqslant \sqrt{\frac{3}{2}}. \tag{45}$$

Since the inhomogeneity of the matter spectrum available for measurements actually refers to N of the order of 60, one can neglect the contribution of the upper limit in the integral, i.e. the value of function $F(u)$, to the leading approximation in $1/N$. Then, the expression for N_{total} is simplified to

$$N_{\text{total}} \approx \frac{3}{4} \left(\frac{1}{z_{\text{in}}^2} - u^2 \ln \frac{1 + u^2 z_{\text{in}}^2}{u^2 z_{\text{in}}^2} \right). \tag{46}$$

Now we can express N_{total} in terms of the experimentally measured r and n_S. We find

$$n_S - 1 = \frac{4(3z^2 + 4z^4 u^2)}{4z^2(1 + u^2 z^2) - 3} = \frac{4(9x_c^2 - z^2)}{3(3x_c^2 - 1)}, \tag{47}$$

and express all of other parameters as follows:

$$z_{\text{in}}^2 = \frac{12r - 3(r - 16)(n_S - 1)}{64}, \tag{48}$$

and

$$x_c^2 = \frac{r}{48}. \tag{49}$$

Making use of the connection between x_c^2 and z^2 according to (42), we get

$$u^2 = \frac{64(-3r + (n_S - 1)(r - 16))}{3(-4r + (n_S - 1)(r - 16))^2}. \tag{50}$$

For the sake of simplicity, introduce the quantity χ

$$\chi = 4r - (n_S - 1)(r - 16), \tag{51}$$

[3] Let us recall, that (42) is valid for chaotic inflation, while in the scenario of new inflation one should change the sign of y^2, i.e. one puts $x_c^2 \approx \frac{4}{9} z^2 (-1 + u^2 z^2)$.

satisfying the condition of $\chi \leqslant r$ equivalent to $u^2 \geqslant 0$, because

$$u^2 = \frac{64}{3}\frac{r-\chi}{\chi^2}.$$

Then, we can easily write down the final expression for N_{total}

$$N_{\text{total}} = \frac{16}{\chi}\left\{1 - \left(1 - \frac{r}{\chi}\right)\ln\left(1 - \frac{\chi}{r}\right)\right\}. \tag{52}$$

The above expression can be "parametrically solved". So, we introduce quantity $\beta \geqslant 0$ according to

$$\chi = r(1-\beta).$$

Then

$$r = \frac{16}{N}\frac{1}{1-\beta}\left\{1 + \frac{\beta}{1-\beta}\,\ln\beta\right\}, \tag{53}$$

and

$$n_S - 1 = -\frac{1-\beta+\beta\ln\beta}{N(1-\beta)^2 - 1 + \beta - \beta\ln\beta}\,(3+\beta). \tag{54}$$

The formula (53) exactly repeats the expression derived in [20, 21] in another notation in the framework of slow-roll approximation, while (54) can match the result of [20, 21], if one neglects subleading terms in the denominator of (54) at $1/N \to 0$.

In the limits of $\beta \to 0$ and $\beta \to 1$ we obtain the reference cases with potentials of form $\lambda\phi^4$ and $m^2\phi^2$, correspondingly. Indeed, up to corrections of the order of $1/N$ as has been suggested in deriving (53) and (54), we find

$$u^2 = \frac{64}{3}\frac{\beta}{r(1-\beta)^2} \to 0 \qquad \text{at } \beta \to 0,$$

hence, the vanishing of the inflaton vacuum expectation value, i.e. nullifying the quadratic term in the potential, while

$$\text{at } \beta \to 1 \qquad z^2 = \frac{3}{64}\,|\chi| = \frac{3}{64}\,r\,|1-\beta| \to 0,$$

that has reduced to zero the quartic term in the potential. In addition,

$$r = \begin{cases} \frac{16}{N}, & \beta = 0, \\ \frac{8}{N}, & \beta = 1, \end{cases} \qquad n_S - 1 = \begin{cases} -\frac{3}{N}, & \beta = 0, \\ -\frac{2}{N}, & \beta = 1, \end{cases} \tag{55}$$

in complete consistency with the consideration of these cases in other approaches.

Generally, the scaling properties of inflation parameters versus $1/N$ are quite complicated because of additional dependence on variable β, which can correlate with the amount of e-folding N. Nevertheless, one can see that at fixed β, the limit of $1/N \to 0$ gives

$$r \sim \frac{1}{N}, \quad x^2 \sim \frac{1}{N}, \quad z^2 \sim \frac{1}{N}, \quad u^2 \sim N,$$

though, actually, the dependence on β could crucially change the asymptotic behavior: for instance, at $\beta \sim \exp[N]$ one gets $r \sim x^2 \sim \exp[-N]$, and $z^2 \sim u^2 \sim 1$. The real situation is clarified after the appropriate analysis of the experimental data set.

III. COMPARING DATA WITH EXPERIMENT

As we have mentioned in the Introduction, a complete analysis of inflation models versus the experimental situation can be found in [20, 21], having presented the evaluation of parameters in the framework of Monte Carlo Markov Chains under the slow-roll approximation of theoretical entries.

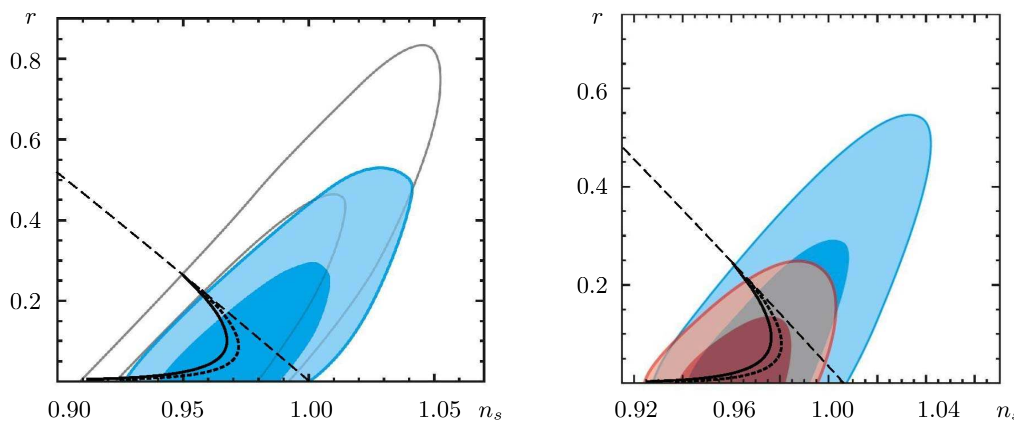

FIG. 1: Data of the WMAP collaboration in the plane of the spectral parameter and the fraction of the tensor term in fluctuations of density: $\{n_S, r\}$, in comparison with theoretical predictions at different values of e-folding $N = 60$ (thick solid line) and $N = 70$ (dotted line), corresponding to the exit of the fluctuation from the event horizon before the end of inflation (see the text). The left panel gives contours representing the WMAP data after 3 years of data taking the confidence levels equal to 1-σ and 2-σ, while the shaded regions give the same confidence levels after 5 years of data sampling. The right panel shows the WMAP data after 5 years of data taking in comparison with further constraints following from BAO and SN experiments.

In the present paper, we will use the function in (52) for constructing the implicit dependence of n_S versus r at fixed N. Theoretical curves are shown in Fig. 1 in the $\{n_S, r\}$-plane (the thick solid line corresponds to $N = 60$, the dotted line does $N = 70$). The dashed line shows $u^2 = 0$, and the region below it corresponds to the actual case of $u^2 > 0$, while the region above it marks $u^2 < 0$, irrelevant to the present work.

The experimental results obtained by the WMAP collaboration after 3 and 5 years of data taking and published in [22] and [23, 24], respectively, are presented in Fig. 1, too. The dark shaded contour gives the region with the 1-σ confidence level, while the shaded contour corresponds to the 2-σ level. One can see, that the theoretical calculations are in a good agreement with the experiment at the appropriate choice of parameters.

From the analysis of data we can obtain quite wide limits of possible values for parameters of the model potential, namely

$$N = 60^{+40}_{-20}, \qquad 25 \leqslant u^2 \leqslant \infty, \tag{56}$$

at the 1-σ level, and

$$N = 60^{+80}_{-27}, \qquad 17 \leqslant u^2 \leqslant \infty, \tag{57}$$

at the 2-σ level in the 3 year data sample by WMAP. The formally infinite vacuum expectation value for the inflaton certainly corresponds to the final value of its mass, as we shall see below. The data acquisition of 5 year sample leads to more strict constraints. So, the above estimates on N with the confidence level of 1-σ transfer to the 2-σ level, as is clearly seen from the figure.

However, the amount of e-folding is, in fact, limited by the actual history of the Universe evolution after inflation [25], so that the analysis leads to the typical value of $N \approx 60$ (see also [13]). In addition, one has to take into account data of other experiments: that on baryonic acoustic oscillations and spacial distribution of galaxies (BAO) [26] as well as on the supernovae Ia (SN) [27, 28, 29, 30]. Such an analysis has been done in [24], and is presented in the right panel of Fig. 1.

Then, the data at the 1-σ level give the constraint on the parameter β in (53) and (54) in the form

$$0.75 \leqslant \beta \leqslant 140. \tag{58}$$

The region of $\beta \leqslant 1$ corresponds to the scenario of chaotic inflation, when the field evolves towards the minimum of potential from large values at the branch of the potential approaching infinity, while $\beta > 1$ describes the scenario of new inflation, when the field "rolls down" to the minimum from small values near the peak at $\phi = 0$ (one refers to

the case of “hilltop” inflation). Indeed, the condition for the critical point (16) during inflation at $y^4 \to 1$ can be approximately written down in the form

$$y^2 \approx \frac{9x^2}{4z^2} - u^2 z^2 = \frac{3}{16}\, r(1-\beta)$$

at $\beta < 1$. So, since $y^2 \sim \phi^2 - v^2$ one can straightforwardly see that $\beta = 1$ just separates the regions of parameters for new and chaotic inflation.

Now let us determine the coupling constant λ. The WMAP, BAO and SN observations give

$$P_S = 2.457^{+0.092}_{-0.093} \cdot 10^{-9}, \tag{59}$$

while

$$\lambda = 8\pi^2 x_c^2 z^4 P_S = \frac{3\pi^2}{2^{13}}\, r \left[4r - (r-16)(n_S - 1)\right]^2 P_S. \tag{60}$$

Therefore, at $N \approx 60$ with (58) we get

$$0 \leqslant \lambda \leqslant 9.7 \cdot 10^{-14}, \tag{61}$$

while the maximum is located at $\beta \approx 35$. The scale of quartic coupling is quite natural, if one takes into account the analysis of $1/N$-expansion during the inflation as performed in [6, 19] and mentioned in Introduction.

It is worth noting, that the product of λu^2 remains finite

$$\lambda u^2 = \frac{\pi^2}{2^7}\, r \left[-3r + (n_S - 1)(r-16)\right] P_S. \tag{62}$$

Moreover, under (58) the square of inflaton mass in vicinity of potential minimum

$$m^2 = \frac{3}{2\pi G}\, \lambda\, u^2$$

takes the values

$$1.03 \cdot 10^{13}\ \text{GeV} \leqslant m \leqslant 1.74 \cdot 10^{13}\ \text{GeV}, \tag{63}$$

maximal at $\beta \approx 7$. Furthermore, since $\mathrm{sign}(m^2) = \mathrm{sign}(v^2) = \mathrm{sign}(u^2)$, the border of the applicability region for the potential is given by the following equation

$$n_S - 1 = -\frac{3r}{16 - r}, \tag{64}$$

which is represented by the dashed line in Fig. 1.

Experimental constraints for dependence of the spectral index on the number of e-folding N in terms of the parameter $dn_S/d\ln N$ are not restrictive, since they give a value compatible with zero at the confidence level of 2-σ, with the quite large uncertainty being greater than the expected value of this parameter in the model under study. Therefore, we do not incorporate it into our estimates.

Thus, we see, that one could extract the mass of the inflaton corresponding to maximal definiteness for all of the potential parameters.

IV. CONCLUSION

Thus, in the present paper we have carried out the analysis of an inflation model with the inflaton potential including both quadratic and quartic terms of self-action. The model has allowed us to consider scenarios of chaotic and new inflation in the framework of the quasiattractor method, which has enabled us to quite elegantly calculate the recently observed inhomogeneity of the cosmic microwave background and distribution of matter in the Universe. We have shown that such a model is consistent with the observational data. One can see, of course, that this model of the potential *parametrically* cannot satisfy all of the experimentally admissible values of n_S and r within the empirical uncertainties (such a potential would not explain the presence of experimental points above the dashed curve in Fig.

1), but these restrictions are not critical within the accuracy of measurements, and the given potential seems to be consistent with the current data.

We have obtained also, that observational data on the inhomogeneity of the Universe corresponds to the time of forming the inflaton fluctuations, when the Universe expands approximately e^{60} times to the end of inflation, which is in agreement with other estimations. We have also precisely enough determined the inflaton mass.

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